DASS: illustration for correlation coefficients

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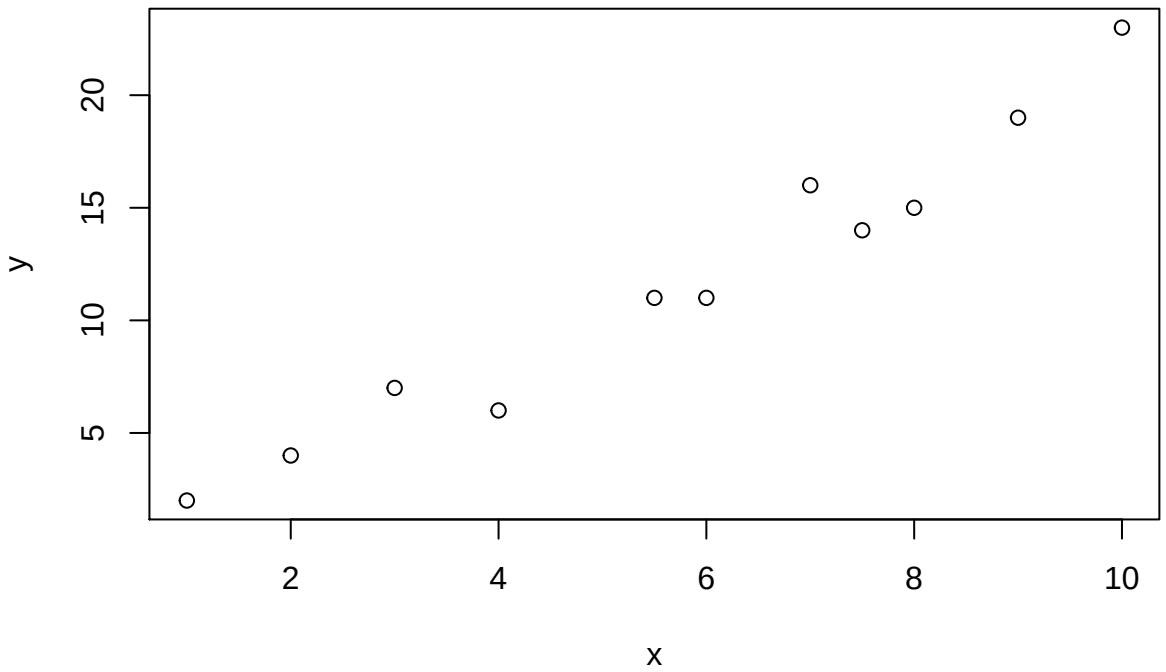
As we discussed, there are two widely used correlation coefficients, a Pearson's one and a Spearman's one. Since the latter is a measure of the rank correlation, it is usually used for variables in an ordinal scale. However, it can be helpful for quantitative variables as well because it is robust (not sensitive to outliers).

Consider two variables: `x` and `y`.

```
x <- c(1, 2, 6, 8, 9, 7, 7.5, 10, 3, 4, 5.5)
y <- c(2, 4, 11, 15, 19, 16, 14, 23, 7, 6, 11)
```

Let's plot a simple scatterplot first:

```
plot(x, y)
```



As we can see, although there are only few points, variables `x` and `y` seem to be positively associated (as `x` increases, `y` increases). We can even say that this association is pretty strong. Let's calculate two correlation coefficients and test their statistical significance.

```
# Pearson's coefficient
cor.test(x, y)
```

```
##
##  Pearson's product-moment correlation
##
## data:  x and y
## t = 13.862, df = 9, p-value = 2.234e-07
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.9124984 0.9942928
```

```
## sample estimates:
##       cor
## 0.9773737
```

What can we see in the output? The correlation coefficient itself is `cor` and here it is 0.977. So, we can conclude that the association between `x` and `y` is positive and very strong (the coefficient is approximately 1). Is it statistically significant at the 5% level of significance? Let us see.

$H_0 : corr(x, y) = 0$ (no linear association between x and y)

This null hypothesis should be rejected at the 5% significance level since p-value < 0.05. So, variables `x` and `y` are associated.

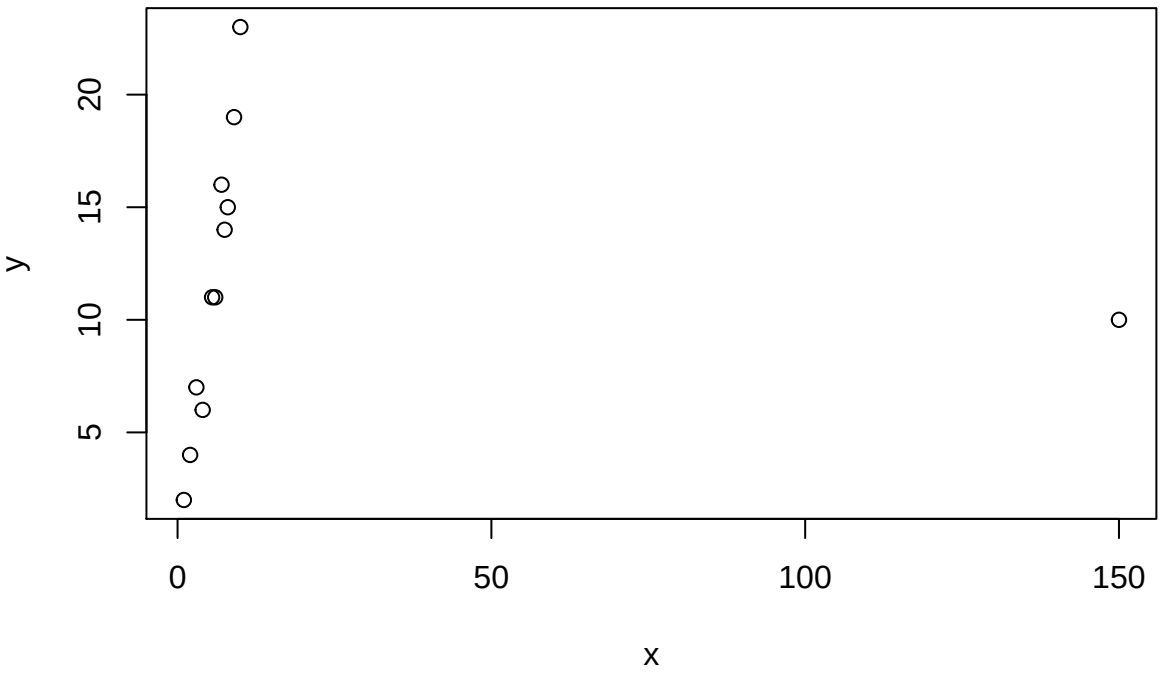
```
# Spearman's coefficient
cor.test(x, y, method = 'spearman')
```

```
## Warning in cor.test.default(x, y, method = "spearman"): Cannot compute
## exact p-value with ties
```

```
##
##  Spearman's rank correlation rho
##
## data:  x and y
## S = 8.5188, p-value = 2.449e-06
## alternative hypothesis: true rho is not equal to 0
## sample estimates:
##       rho
## 0.9612781
```

Here we also get a very high positive coefficient (0.96). Now let us add an outlier, a non-typical observation to our data, a point (150, 10).

```
x <- c(1, 2, 6, 8, 9, 7, 7.5, 10, 3, 4, 5.5, 150)
y <- c(2, 4, 11, 15, 19, 16, 14, 23, 7, 6, 11, 10)
plot(x, y)
```



It seems that this point can spoil everything! We can calculate correlation coefficient for updated variables:

```r
cor.test(x, y)
```

```
##
##  Pearson's product-moment correlation
##
## data:  x and y
## t = -0.033164, df = 10, p-value = 0.9742
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  -0.5808924  0.5668262
## sample estimates:
##         cor
## -0.01048683
```

A Pearson's correlation coefficient has broken down! Now it is negative, very small by absolute value and, what is more, insignificant! This coefficient is very sensitive to outliers, so here it "reacts" on a non-typical point in a very dramatic way. Now let's look at a Spearman's coefficient:

```r
cor.test(x, y, method = 'spearman')
```

```
## Warning in cor.test.default(x, y, method = "spearman"): Cannot compute
## exact p-value with ties
```

```
##
##  Spearman's rank correlation rho
##
## data:  x and y
## S = 64.613, p-value = 0.003127
## alternative hypothesis: true rho is not equal to 0
## sample estimates:
##       rho
## 0.7740817
```

Magic! This coefficient has not undergone serious changes, it is still positive and high. Besides, it is significant at the 5% significance level. So, with the help of this illustration we made sure that a Spearman's correlation coefficient is more robust than Pearson's one.