Math in Moscow, 2014-15 academic year
Ordinary differential equations (http://math-info.hse.ru/s14/12)
Exercises for lesson 7 (04/02/2015)
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Problem 1. Consider equation $(\dot{x}, \dot{y}) = w(x, y)$. Find new coordinates (u, v), taking this equation to the form

$$\dot{u} = 1, \quad \dot{v} = 0$$

near point P:

(a) $w = (1, 2)$, $P = (0, 0)$;
(b) $w = (x, 2y)$, $P = (1, 1)$;
(c) $w = (2x, -y)$, $P = (-2, 2)$;
(d) (*) $w = (y, -x)$, $P = (1, 0)$;

Problem 2. Rewrite the following systems in polar coordinates. Plot its phase portraits in new and old coordinates.

(a) $\dot{x} = -y, \quad \dot{y} = x$;
(b) $\dot{x} = y, \quad \dot{y} = -x$;
(c) $\dot{x} = x - y, \quad \dot{y} = x + y$;