Math in Moscow, 2014-15 academic year
Ordinary differential equations (http://math-info.hse.ru/s14/12)
Exercises for lesson 6 (03/19/2015)
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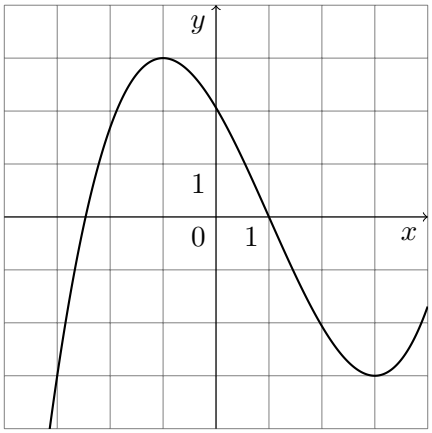
Problem 1. Consider function $y = f(x)$ which graph is shown on the picture. Sketch graphs of the following functions on the same axis. (Use chain rule to investigate the behaviour of the function near the border of its domain.)



(a) $\sqrt{f(x)}$;
(b) $\sqrt{f(x)-2}$;
(c) $\sqrt{f(x)-3}$;
(d) $\sqrt{f(x)-4}$;
(e) $\sqrt{f(x)+1}$;
(f) $\sqrt{f(x)+2}$;
(g) $\sqrt{f(x)+3}$;
(h) $\sqrt{f(x)+4}$.

Problem 2. Consider equation $\ddot{x} = f'(x)$, where $f(x)$ is a function from the previous problem.

(a) Find potential energy in terms of f. Sketch its graph.
(b) Write down the function of full energy.
(c) Sketch some level curves of the function of full energy.
(d) Plot the phase portrait. Depict all equilibria. Which of them are stable and which are unstable?

Problem 3. Plot phase portraits of the following equations:

(a) $\ddot{x} = x^2 - 1$;
(b) $\ddot{x} = 1 - x^2$;
(c) $\ddot{x} = 4x^3 - 4x$
(d) $\ddot{x} = -4x^3 + 4x$