

Math in Moscow, 2014-15 academic year**Ordinary differential equations** (<http://math-info.hse.ru/s14/12>)**Exercises for lesson 2 (02/19/2015)***Ilya Schurov*

Problem 1. Construct Euler's approximations for the solution of equation $\dot{x} = x$ with initial condition $x(0) = 1$ with $\Delta t = 1/n$ for integer n on the segment $[0, 1]$. Find a limit of the value of the approximation at $t = 1$ as $n \rightarrow \infty$.

Problem 2. Find all solutions of an equation $\dot{x} = \sqrt{x}$ with initial condition $x(0) = 0$.

Problem 3. Find all solutions of an equation in form $y' = y(x)$. Sketch corresponding direction field and integral curve.

(a) $y' = y/x;$

(b) $y' = -x/y;$

(c) $y' = xy;$

Problem 4. Solve the following equations (it is not needed to express the solution as a functions of x), find a solution with given initial condition (if present):

(a) $\dot{x} = 2x + t, \quad x(0) = -1/4.$

(c) $y' = -xe^y;$

(b) $(x^2 + 4)y' = 2xy;$

Remark 1. *Sometimes you have to make a substitution to solve an equation.*

Problem 5. Solve the following equations (it is not needed to express the solution as a functions of x):

(a) $y' = \frac{y(1+xy)}{x(1-xy)};$

(c) $y' = \sin(x + y);$

(b) $y' = -\frac{x+y+1}{4x+4y+10};$

(d) $y' = \sqrt{4x + 2y - 1}.$