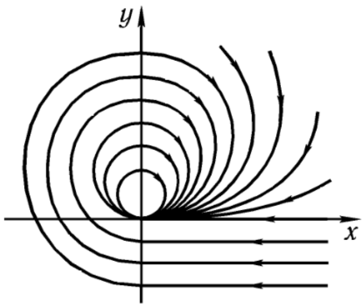

Figure 1: Figure for problem 2

Math in Moscow, 2014-15 academic year
Ordinary differential equations (http://math-info.hse.ru/s14/12)
Exercises for lesson 11 (04/30/2015)
Ilya Schurov

Problem 1. [1] Use the definitions to investigate the stability of all equilibria of the following equations and systems. Decide are they Lyapunov stable, asymptotically stable, Lyapunov unstable?

(a) $\dot{x} = 0;$

(b) $\begin{cases} \dot{x} = 0, \\ \dot{y} = y \end{cases}$

(c) $\begin{cases} \dot{x} = y, \\ \dot{y} = -x \end{cases}$

(d) $\begin{cases} \dot{x} = x^2 \\ \dot{y} = -y \end{cases}$

(e) $\begin{cases} \dot{x} = y + x(1 - x^2 - y^2) \\ \dot{y} = -x + y(1 - x^2 - y^2) \end{cases}$

(f) $\begin{cases} \dot{x} = y - x(x^2 + y^2) \\ \dot{y} = -x - y(x^2 + y^2) \end{cases}$

Problem 2. [2] Fig. 1 depicts the trajectories of the system

$$\begin{cases} \dot{x} = f(x, y), \\ \dot{y} = g(x, y), \end{cases}$$

where f, g, f'_x, f'_y, g'_x, g'_y are continuous functions. What can you say about limit behaviour of solutions as $t \to +\infty$? Is the origin asymptotically stable? Lyapunov stable?

Problem 3. Consider equation $\dot{x} = v(x)$, $x(t) \in \mathbb{R}^n$, $v(0) = 0$. Assume there exists a solution $x(t)$ such that $x(0) \neq 0$ and $\lim_{t\to-\infty} x(t) = 0$. What cam you say about the stability of equlibria $x = 0$?

References

[1] V. I. Arnold. Ordinary differential equations.
[2] A. F. Filippov. Problems in differential equations