

**Math in Moscow, 2014-15 academic year****Ordinary differential equations** (<http://math-info.hse.ru/s14/12>)**Assignment ODE-7 (To be returned 04/16/2015)***Ilya Schurov***1** (3 points each). Solve the following ODEs:

(a).  $xy' + (x + 1)y = 3x^2e^{-x}$ ;

(b).  $2x(x^2 + y)dx = dy$ ;

(c).  $(2e^y - x)y' = 1$ ;

(d).  $y' = \frac{y}{3x - y^2}$ .

**2** (2). Consider linear operator  $D = \frac{d^2}{dt^2} - 5\frac{d}{dt} + 6$ . Decompose  $D$  as  $(\frac{d}{dt} - \lambda_1) \circ (\frac{d}{dt} - \lambda_2)$ .**3** (2 each). Using method of undetermined coefficients, find all solutions of equation. (Hint: to find a particular solution, first find appropriate space of quasipolynomials that can pretend to be a solution. Recall the properties of operator  $\frac{d}{dt} - \lambda$  when it acts on quasipolynomials discussed at the lecture.)

(a).  $\ddot{x} - 5\dot{x} + 6x = 0$

(b).  $\ddot{x} - 5\dot{x} + 6x = e^t$

(c).  $\ddot{x} - 5\dot{x} + 6x = t^2e^t$

(d).  $\ddot{x} - 5\dot{x} + 6x = e^{2t}$

(e).  $\ddot{x} - 5\dot{x} + 6x = t^2e^{2t}$

(f).  $\ddot{x} - 5\dot{x} + 6x = \sin t$

**4** (3). Consider equation  $\ddot{x} + p\dot{x} + qx = f(t)$ , where  $p$  and  $q$  are real numbers and  $f(t)$  is a real-valued function. Assume that  $x(t)$  is a complex solution of this equation. Prove that  $\overline{x(t)}$  (complex conjugacy) and  $\operatorname{Re} x(t)$  are also solutions of this equation. (Hint: the space of solutions of nonhomogenous equation is an affine subspace and  $\operatorname{Re} x = \frac{x + \overline{x}}{2}$ .)**5** (2). Consider linear operator  $D = \frac{d^2}{dt^2} + 4$ . Decompose  $D$  as  $(\frac{d}{dt} - \lambda_1) \circ (\frac{d}{dt} - \lambda_2)$  (for complex  $\lambda$ 's).**6** (2 each). Find all *real* solutions of the following equations. (Hint: first find all complex solutions, then use result of problem 4.)

(a).  $\ddot{x} + 4x = 0$ ;

(b).  $\ddot{x} + 4x = \sin t$ ;

(c).  $\ddot{x} + 4x = t^2 \sin t$ ;

(d).  $\ddot{x} + 4x = \sin 2t$ ;

(e).  $\ddot{x} + 4x = t^2 \sin 2t$ ;

(f).  $\ddot{x} + 4x = e^t$ .