Math in Moscow, 2014-15 academic year
Ordinary differential equations (http://math-info.hse.ru/s14/12)
Assignment ODE-7 (To be returned 04/16/2015)
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1 (3 points each). Solve the following ODEs:
(a). $xy' + (x+1)y = 3x^2e^{-x}$;
(b). $2x(x^2+y)dx = dy$;
(c). $(2e^y - x)y' = 1$;
(d). $y' = \frac{y}{3x-y^2}$.

2 (2). Consider linear operator $D = \frac{d^2}{dt^2} - 5\frac{d}{dt} + 6$. Decompose D as $(\frac{d}{dt} - \lambda_1) \circ (\frac{d}{dt} - \lambda_2)$.

3 (2 each). Using method of undetermined coefficients, find all solutions of equation. (Hint: to find a particular solution, first find appropriate space of quasipolynomials that can pretend to be a solution. Recall the properties of operator $\frac{d}{dt} - \lambda$ when it acts on quasipolynomials discussed at the lecture.)
(a). $\ddot{x} - 5\dot{x} + 6x = 0$
(b). $\ddot{x} - 5\dot{x} + 6x = e^t$
(c). $\ddot{x} - 5\dot{x} + 6x = t^2e^t$
(d). $\ddot{x} - 5\dot{x} + 6x = e^{2t}$
(e). $\ddot{x} - 5\dot{x} + 6x = t^2e^{2t}$
(f). $\ddot{x} - 5\dot{x} + 6x = \sin t$

4 (3). Consider equation $\ddot{x} + p\dot{x} + qx = f(t)$, where p and q are real numbers and $f(t)$ is a real-valued function. Assume that $x(t)$ is a complex solution of this equation. Prove that $\overline{x(t)}$ (complex conjugacy) and $\operatorname{Re} x(t)$ are also solutions of this equation. (Hint: the space of solutions of nonhomogenious equation is an affine subspace and $\operatorname{Re} x = \frac{x+\overline{x}}{2}$.)

5 (2). Consider linear operator $D = \frac{d^2}{dt^2} + 4$. Decompose D as $(\frac{d}{dt} - \lambda_1) \circ (\frac{d}{dt} - \lambda_2)$ (for complex λ's).

6 (2 each). Find all *real* solutions of the following equations. (Hint: first find all complex solutions, then use result of problem 4.)
(a). $\ddot{x} + 4x = 0$;
(b). $\ddot{x} + 4x = \sin t$;
(c). $\ddot{x} + 4x = t^2 \sin t$;
(d). $\ddot{x} + 4x = \sin 2t$;
(e). $\ddot{x} + 4x = t^2 \sin 2t$;
(f). $\ddot{x} + 4x = e^t$.