1 (3 points each). Solve the following ODEs:
(a). \(xy' + (x + 1)y = 3x^2e^{-x}\);
(b). \(2x(x^2 + y)dx = dy\);
(c). \((2e^y - x)y' = 1\);
(d). \(y' = \frac{y}{3x-y^2}\).

2 (2). Consider linear operator \(D = \frac{d^2}{dt^2} - 5\frac{d}{dt} + 6\). Decompose \(D\) as \((\frac{d}{dt} - \lambda_1) \circ (\frac{d}{dt} - \lambda_2)\).

3 (2 each). Using method of undetermined coefficients, find all solutions of equation.
(Hint: to find a particular solution, first find appropriate space of quasipolynomials that can pretend to be a solution. Recall the properties of operator \(\frac{d}{dt} - \lambda\) when it acts on quasipolynomials discussed at the lecture.)
(a). \(\ddot{x} - 5\dot{x} + 6x = 0\)
(b). \(\ddot{x} - 5\dot{x} + 6x = e^t\)
(c). \(\ddot{x} - 5\dot{x} + 6x = t^2e^t\)
(d). \(\ddot{x} - 5\dot{x} + 6x = e^{2t}\)
(e). \(\ddot{x} - 5\dot{x} + 6x = t^2e^{2t}\)
(f). \(\ddot{x} - 5\dot{x} + 6x = \sin t\)

4 (3). Consider equation \(\ddot{x} + px + qx = f(t)\), where \(p\) and \(q\) are real numbers and \(f(t)\) is a real-valued function. Assume that \(x(t)\) is a complex solution of this equation. Prove that \(\bar{x}(t)\) (complex conjugacy) and \(\text{Re}\ x(t)\) are also solutions of this equation. (Hint: the space of solutions of nonhomogenous equation is an affine subspace and \(\text{Re}\ x = \frac{x + \bar{x}}{2}\).)

5 (2). Consider linear operator \(D = \frac{d^2}{dt^2} + 4\). Decompose \(D\) as \((\frac{d}{dt} - \lambda_1) \circ (\frac{d}{dt} - \lambda_2)\) (for complex \(\lambda_1\)'s).

6 (2 each). Find all real solutions of the following equations. (Hint: first find all complex solutions, then use result of problem 4)
(a). \(\ddot{x} + 4x = 0\);
(b). \(\ddot{x} + 4x = \sin t\);
(c). \(\ddot{x} + 4x = t^2\sin t\);
(d). \(\ddot{x} + 4x = \sin 2t\);
(e). \(\ddot{x} + 4x = t^2\sin 2t\);
(f). \(\ddot{x} + 4x = e^t\).