

Math in Moscow, 2014-15 academic year**Ordinary differential equations** (<http://math-info.hse.ru/s14/12>)**Assignment ODE-6 (To be returned 04/09/2015)***Ilya Schurov***1** (1 + 1 + 3 + 2 + 2). Consider equation

$$\ddot{x} = x^3 - x^2 - 2x$$

- (a). Write down the function of full energy.
- (b). Plot the graph of potential energy.
- (c). Plot the phase portrait. Depict all equilibria. Which of them are stable and which are unstable?
- (d). For which initial conditions (x_0, \dot{x}_0) the solution is periodic?
- (e). For which initial conditions the solution has limit as $t \rightarrow +\infty$?

2 (4). Consider equation $(\dot{x}, \dot{y}) = (2x, y + 1)$. Find new coordinates (u, v) , taking this equation to the form

$$\dot{u} = 1, \quad \dot{v} = 0$$

near point $P = (0, 0)$.**3** (4 + 6). Rewrite the following system in polar coordinates. Plot phase portrait in the initial and new coordinates:

$$(a). \begin{cases} \dot{x} = y + x \\ \dot{y} = -x + y \end{cases} \qquad (b). \begin{cases} \dot{x} = y + x(1 - x^2 - y^2) \\ \dot{y} = -x + y(1 - x^2 - y^2) \end{cases}$$