Math in Moscow, 2014-15 academic year
Ordinary differential equations (http://math-info.hse.ru/s14/12)
Assignment ODE-5 (To be returned 20/03/2015)
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1 $(2+2+2+2)$. For the following systems of ODE find some nonconstant globally defined continuous first integral or prove that it does not exists.
(a). $\dot{x} = \sin(x+y), \quad \dot{y} = \cos(x+y+z), \quad \dot{z} = 0;$
(b). $\dot{x} = -y, \quad \dot{y} = x, \quad \dot{z} = \sin(x^2+y^2+z^2);$
(c). $\dot{x} = x, \quad \dot{y} = 2y, \quad \dot{z} = -3z.$
(d). $\dot{x} = x, \quad \dot{y} = 2y, \quad \dot{z} = 3z.$

2 (4). For which real k there exists nonconstant globally defined continuous first integral for the system:

$$\dot{x} = x, \quad \dot{y} = ky.$$

3 $(1+1+1+2)$. Let f, g be smooth functions $\mathbb{R}^n \to R$ and v, w be smooth vector fields on $\mathbb{R}^n$. Prove the following properties of Lie derivative:
(a). $L_v(f+g) = L_v f + L_v g;$
(b). $L_{v+w} f = L_v f + L_w f;$
(c). $L_{fv} g = f L_v g;$
(d). $L_v(fg) = f L_v g + g L_v f.$

4 $(2+1)$. Let $f\colon \mathbb{R} \to \mathbb{R}$ and $G\colon \mathbb{R}^n \to \mathbb{R}$ be smooth functions and v be vector field. Find $L_v f \circ G$. Use this fact to prove that if G is a first integral of some ODE then $f \circ G$ is also first integral of that ODE.