Math in Moscow, 2014-15 academic year

Ordinary differential equations (http://math-info.hse.ru/s14/12)

Assignment ODE-3 (To be returned 5/03/2015)

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1 \((2 + 2 + 3)\). Draw vector field of the system. Find equation for phase curves. Draw phase portrait. Depict all singular points (i.e. points where the vector field vanishes).

(a). \(\dot{x} = x^3 - x, \quad \dot{y} = 0\);
(b). \(\dot{x} = y, \quad \dot{y} = 1\);
(c). \(\dot{x} = 3x, \quad \dot{y} = 2y\).

Definition 1. An \textit{isocline} of a vector field is a curve through points at which the slope of a vector will always be the same. Isoclines are useful when you are drawing a phase portrait.

2 \((2 + 1 + 3 + 3)\). Consider Lotka-Volterra equation:

\[
\dot{x} = kx - axy, \quad \dot{y} = -ly + bxy, \tag{1}
\]

where \(k, a, b, l > 0, x \geq 0, y \geq 0\).

This equation describes the dynamics of biological systems in which two species interact, one a predator and one its prey.

Let \(k = a = b = l = 1\).

(a). Draw the vector field.
(b). Draw vertical (horizontal) isoclines: curves through points at which all vectors are vertical (horizontal).
(c). Find an equation of the phase curves of system \((1)\) by solving corresponding non-autonomous differential equation.
(d). Draw the phase portrait.