

**Math in Moscow, 2014-15 academic year****Ordinary differential equations** (<http://math-info.hse.ru/s14/12>)**Assignment ODE-3 (To be returned 5/03/2015)***Ilya Schurov*

**1** (2 + 2 + 3). Draw vector field of the system. Find equation for phase curves. Draw phase portrait. Depict all singular points (i.e. points where the vector field vanishes).

(a).  $\dot{x} = x^3 - x, \quad \dot{y} = 0;$

(b).  $\dot{x} = y, \quad \dot{y} = 1;$

(c).  $\dot{x} = 3x, \quad \dot{y} = 2y.$

**Definition 1.** An *isocline* of a vector field is a curve through points at which the slope of a vector will always be the same. Isoclines are useful when you are drawing a phase portrait.

**2** (2 + 1 + 3 + 3). Consider Lotka-Volterra equation:

$$\dot{x} = kx - axy, \quad \dot{y} = -ly + bxy, \tag{1}$$

where  $k, a, b, l > 0, x \geq 0, y \geq 0$ .

This equation describes the dynamics of biological systems in which two species interact, one a predator and one its prey.

Let  $k = a = b = l = 1$ .

(a). Draw the vector field.

(b). Draw vertical (horizontal) isoclines: curves through points at which all vectors are vertical (horizontal).

(c). Find an equation of the phase curves of system (1) by solving corresponding non-autonomous differential equation.

(d). Draw the phase portrait.