Math in Moscow, 2014-15 academic year
Ordinary differential equations (http://math-info.hse.ru/s14/12)
Assignment ODE-2 (To be returned 02/26/2015)
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1 (each part is 2 points). Find all solutions of an equation in form $y = y(x)$. Describe explicitly the domain of each solution. Plot some integral curves.

(a) $y' = y/(2x)$; (b) $y' = -y/x$; (c) $y' = -4x/y$; (d) $y' = -xy$. .

2 (each part is 3 points). Solve the following equations (it is not needed to express the solution as a functions of x, the answer in implicit form (but without integrals) is acceptable):

(a) $xy' + y = y^2$;

(b) $(x + 2y)y' = 1$. Find a solution with initial condition $y(0) = -1$; (Hint: use appropriate substitution).

(c) $xy' = x + y$ (hint: consider substitution $z = y/x$);

(d) $y' = \sqrt[3]{y}$. Find *all* solutions with initial condition $y(0) = 0$. (This problem is the same as 3c from assignment 1, you can skip it if you believe that you solved that problem correctly.)

3 $(3+1+4)$. Denote by $x(t)$ the size of fish population at moment t, and assume that its normal growth rate is given by the formula $(1-x)x$. Coefficient $(1-x)$ here is due to the fact that resources (e.g. food) are limited: if the size of the population is small, then $1 - x \approx 1$ and we have almost Malthusian exponential growth, but as x increases relative reproduction rate decreases. Assume that we additionally fish out some fixed amount of fish in every unit of time — denote this value by $c > 0$ (fishing quota). So the equation for x is the following:

$$\dot{x} = x - x^2 - c$$

(a) For which pairs of value of the fishing quota and the initial population size the population survives forever?

(b) What is the maximal value c_{max} of c for which the population survives forever if the initial population is big enough?

(c) Sketch the integral curves of corresponding differential equation for different values of c. (No exact solution needed.) Consider three cases: $0 < c < c_{max}$, $c = c_{max}$ and $c > c_{max}$.

4 (4). The following Lemma was discussed on the lecture. Let f be C^1-smooth function (i.e. f' exists and is continuous) and $f(x_0) = 0$. Then there exists constant C and interval $(x_1, x_2) \ni x_0$ such that $|f(x)| < C|x - x_0|$ for all $x \in [x_1, x_2]$.

On the lecture, we proved this Lemma using integration. Prove it using mean value theorem instead of integration.