1 (each part is 2 points). Find all solutions of an equation in form \( y = y(x) \). Describe explicitly the domain of each solution. Plot some integral curves.

(a) \( y' = y/(2x) \);
(b) \( y' = -y/x \);
(c) \( y' = -4x/y \);
(d) \( y' = -xy \).

2 (each part is 3 points). Solve the following equations (it is not needed to express the solution as a functions of \( x \), the answer in implicit form (but without integrals) is acceptable):

(a) \( xy' + y = y^2 \);
(b) \( (x + 2y)y' = 1 \). Find a solution with initial condition \( y(0) = -1 \); (Hint: use appropriate substitution).
(c) \( xy' = x + y \) (hint: consider substitution \( z = y/x \));
(d) \( y' = \sqrt{y} \). Find all solutions with initial condition \( y(0) = 0 \). (This problem is the same as 3c from assignment 1, you can skip it if you believe that you solved that problem correctly.)

3 (3+1+4). Denote by \( x(t) \) the size of fish population at moment \( t \), and assume that its normal growth rate is given by the formula \( (1 - x)x \). Coefficient \( (1 - x) \) here is due to the fact that resources (e.g. food) are limited: if the size of the population is small, then \( 1 - x \approx 1 \) and we have almost Malthusian exponential growth, but as \( x \) increases relative reproduction rate decreases. Assume that we additionally fish out some fixed amount of fish in every unit of time — denote this value by \( c > 0 \) (fishing quota). So the equation for \( x \) is the following:

\[
\dot{x} = x - x^2 - c
\]

(a) For which pairs of value of the fishing quota and the initial population size the population survives forever?

(b) What is the maximal value \( c_{\text{max}} \) of \( c \) for which the population survives forever if the initial population is big enough?

(c) Sketch the integral curves of corresponding differential equation for different values of \( c \). (No exact solution needed.) Consider three cases: \( 0 < c < c_{\text{max}} \), \( c = c_{\text{max}} \) and \( c > c_{\text{max}} \).

4 (4). The following Lemma was discussed on the lecture. Let \( f \) be \( C^1 \)-smooth function (i.e. \( f' \) exists and is continuous) and \( f(x_0) = 0 \). Then there exists constant \( C \) and interval \((x_1, x_2) \ni x_0 \) such that \( |f(x)| < C|x - x_0| \) for all \( x \in [x_1, x_2] \).

On the lecture, we proved this Lemma using integration. Prove it using mean value theorem instead of integration.