Math in Moscow, 2014-15 academic year

Ordinary differential equations (http://math-info.hse.ru/s14/12)

Assignment ODE-1 (To be returned 02/19/2015)

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1 (each part is 2 points). Sketch the direction fields and the integral curves of the following differential equations. (Exact solutions are not needed.)

(a) $\dot{x} = 5t$; (b) $\dot{x} = \frac{t}{x}$; (c) $\dot{x} = -\frac{4}{x}$; (d) $\dot{x} = -\frac{x}{t}$; (e) $\dot{x} = \sin x$; (f) $\dot{x} = xt$.

2 (2 + 2). Solve the following differential equations.

(a) $\dot{x} = \sin(t)$; (b) $\dot{x} = \sqrt{t}$.

3 (2 + 2 + 4). Find all solutions of the following differential equations.

(a) $\dot{x} = x^2$;
(b) $\dot{x} = x \log x$, $x > 0$;
(c) $\dot{x} = \sqrt{x}$ (hint: how many solutions with initial condition $x(0) = 0$ can you find?)

4 (2 + 2 + 1 + 3 + 1 + 1 + 1). Consider differential equation

$$\dot{x} = x^2 - 1$$ (1)

(a) Draw the direction field for this equation.
(b) Sketch the integral curves (graphs of the solutions).
(c) Are there any constant solutions ($x(t) \equiv x_0$)? If yes, find them.
(d) Solve the initial value problem $x(0) = x_0$. (I.e. find a solution of (1) which satisfies the initial condition; $x_0$ is arbitrary but fixed number.)
(e) Draw integral curves for initial conditions $x(0) = -2$, $x(0) = 0$, $x(0) = 7$.
(f) What can you say about limit behaviour of the solutions for $t \to +\infty$ and $t \to -\infty$?
(g) What can you say about vertical asymptotes of the solutions?

5 (2). Consider radioactive decay law: the amount of a radioactive substance which decays in a unit of time is proportional to the current amount of substance. Assume that after 30 days 50% of the substance decay. How long does it take to decay so only 1% of the original amount remain?

Bonus

6 (5). For any $\varepsilon > 0$ construct a 1-st order differential equation with smooth (and everywhere defined) right-hand part that has no solutions defined on any interval longer than $\varepsilon$.

1 Gives bonus points; can be skipped safely