

**Math in Moscow, 2014-15 academic year****Ordinary differential equations** (<http://math-info.hse.ru/s14/12>)**Assignment ODE-1 (To be returned 02/19/2015)***Ilya Schurov*

**1** (each part is 2 points). Sketch the direction fields and the integral curves of the following differential equations. (Exact solutions are not needed.)

(a)  $\dot{x} = 5t$ ; (b)  $\dot{x} = \frac{x}{t}$ ; (c)  $\dot{x} = -\frac{t}{x}$ ; (d)  $\dot{x} = -\frac{x}{t}$ ; (e)  $\dot{x} = \sin x$ ; (f)  $\dot{x} = xt$ .

**2** (2 + 2). Solve the following differential equations.

(a)  $\dot{x} = \sin(t)$ ; (b)  $\dot{x} = \sqrt{t}$ .

**3** (2 + 2 + 4). Find all solutions of the following differential equations.

(a)  $\dot{x} = x^3$ ;

(b)  $\dot{x} = x \log x$ ,  $x > 0$ ;

(c)  $\dot{x} = \sqrt[3]{x}$  (hint: how many solutions with initial condition  $x(0) = 0$  can you find?)

**4** (2 + 2 + 1 + 3 + 1 + 1 + 1). Consider differential equation

$$\dot{x} = x^2 - 1 \tag{1}$$

(a) Draw the direction field for this equation.

(b) Sketch the integral curves (graphs of the solutions).

(c) Are there any constant solutions ( $x(t) \equiv x_0$ )? If yes, find them.

(d) Solve the initial value problem  $x(0) = x_0$ . (I.e. find a solution of (1) which satisfies the initial condition;  $x_0$  is arbitrary but fixed number.)

(e) Draw integral curves for initial conditions  $x(0) = -2$ ,  $x(0) = 0$ ,  $x(0) = 7$ .

(f) What can you say about limit behaviour of the solutions for  $t \rightarrow +\infty$  and  $t \rightarrow -\infty$ ?

(g) What can you say about vertical asymptotes of the solutions?

**5** (2). Consider radioactive decay law: the amount of a radioactive substance which decays in a unit of time is proportional to the current amount of substance. Assume that after 30 days 50% of the substance decay. How long does it take to decay so only 1% of the original amount remain?

**Bonus<sup>1</sup>**

**6** (5). For any  $\varepsilon > 0$  construct a 1-st order differential equation with smooth (and everywhere defined) right-hand part that has no solutions defined on any interval longer than  $\varepsilon$ .

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<sup>1</sup>Gives bonus points; can be skipped safely